



The Effect of Variable Lead Time in an Integrated Vendor-Buyer Inventory System with Transportation Cost

W.Ritha and J.C.Eveline

Department of Mathematics, Holy Cross College (Autonomous), Tiruchirappalli-620002, India.

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ABSTRACT

This paper contemplates a single vendor single buyer integrated production inventory problem with stochastic demand and transportation cost. Instead of constant lead time, it is assumed to be proportional to the size of vendor's batch in each cycle. That is, lead time is composed of a lot size dependent run time and constant delay times such as moving, waiting and setup times. Transportation takes a crucial part in the manipulation of supply chain. The operation of transportation determines the efficiency of moving products. The problem is to find the number of shipments m , shipment size Q , safety factor k and the reorder point r . The objective is to minimize the joint total relevant cost incurred in the supply chain. A solution procedure is mentioned for solving the proposed model. Finally a numerical example is provided to illustrate the proposed model.

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Introduction

In contemporary competitive business environment, it is imperative for businesses to continuously work on improving the performance of supply chains. Consequently, integrated supply chain decisions and coordination across supply chains are frequently sought for improving performance of supply chains. Integration of vendor's and buyer's individual problems in a supply chain has been a point of interest of many supply chain researchers during the past few decades. This is because integrated policy has the ability to offer customers shorter lead time and lower inventory cost. It also helps to determine problem areas along the process enabling businesses to take decisions action and further reduce cost to improve the final price. Improved customer satisfaction and loyalty is a byproduct of an integrated supply chain because the end customers experience improved on-time delivery. It also makes the system as a whole more robust enabling both the vendor and the buyer to be more flexible in dealing with sudden disruptions.

Supply chain management (SCM) is a systematic progression in which an organization manages the flows of products, services, money, etc. The aim is to obtain maximum profit with minimum costing as well as fulfilling the customer's demand. The single-vendor single-buyer cooperative production inventory model received a lot of interest in recent years by several researchers. The model facing the customer is how much to order in each purchase order. On the other hand, the model facing the seller is to make a decision the final production batch size and the most economical number of shipments in which the whole order quantity to consumer will be supplied. Therefore, an integrated inventory rule is useful to decide the economic order quantity and shipment policy.

Because of global supply chain, the transportation during shipment of products becomes a major challenge among all players of supply chain. Due to this matter, transportation cost should be included in the total cost to calculate the whole supply chain cost. The key element in the supply chain is transportation system which joints the separated activities. The progress in techniques and management principles improves the moving load, delivery speed, service quality, operation costs, the usage of facilities and energy saving. A good transport system in supply chain activities could provide better supply chain efficiency, reduce operation cost and promote service quality. Without well developed transportation systems, supply chain could not bring it's advantages into full play.

The rest of the paper is organized as follows: Section 2 presents the literature review. Section 3 provides fundamental notations and assumptions. Section 4 describes the model formulation. Section 5 illustrates a numerical example. The paper concludes in Section 6. A list of references is also provided.

2. Literature Review

Nowadays, companies can no longer compete solely as individual entities in the constantly changing business world. Globalization of market and increased competition force organizations to rely on effective supply chain to improve their overall performance. Goyal [10] considered a single vendor single buyer model to optimize the joint total cost of the system. Banerjee [3] enhanced the model by incorporating a finite production rate with a lot-for-lot policy for the vendor. Goyal [11] described a more general joint economic lot sizing model by relaxing Banerjee's lot-for-lot assumption. Lu [24] discovered the optimal production and shipment policies when the shipment sizes are equal.

Tele:

E-mail address: ritha_prakash@yahoo.co.in

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He relieved the assumption in Goyal [11] where full must be achieved before shipments. Goyal [12] found an expression for the optimal first shipment sizes as a function of the number of shipments by establishing a model where successive shipment sizes increase by a ratio equal to the production rate divided by the demand rate. Further, Hill [15] extended the model by considering the geometric growth factor as a decision variable. Finally, Hill [16] enriched an optimal policy which was a general form of the policy by Goyal [12] who assumed equal-sized shipments between the vendor and the buyer. Later, Goyal [13] recommended a simple procedure for determining the optimal ordering policy. Most of the researches on the joint vendor-buyer problem assumed that the demand is deterministic. When the demand during the cycle period is stochastic, we must consider the safety stock in order to satisfy service level. Liao and Shyu [23] proposed a probabilistic inventory model considering lead time uncertainty. Ben-Daya and Hariga [5] suggested a solution procedure for solving an integrated single vendor single-buyer model with stochastic demand and variable lead time. Ouyang, Wu, and Ho [27] and Jauhari, Pujawan, Wiratno, and Priyandari [19] extended the model under integrated system of single-vendor and single-buyer. Cardenas-Barron, Teng, Trevino-Garza, Wee, and Lou [8] elucidated an improved algorithm and solution procedure for the three-layer supply chain production inventory model.

Freight rate costs are usually computed based on shipping weight and distance in this paper. The theoretical models with transportation and inventory costs were initiated by Baumol and Vinod [4]. Subsequently, Langley [21] incorporated the actual motor carrier freight rates function into lot sizing decision using enumeration technique. Currently, Mendoza and Ventura [25] presented an algorithm based on grossly simplified freight rate structure (using either a constant charge per truckload (TL) or a constant cost per unit for less-than-truckload (LTL) shipments). He, Hu, and Guo [14] developed an algorithm to derive the optimal purchase quantity using actual freight rates without considering purchase quantity discounts. Actual shipping decision falls into three categories: (1) TL shipping quantities delivery, (2) delivery over the TL, and (3) delivery at LTL (Swenseth & Godfrey, [32]). Darwish [9] enhanced the model by considering freight rate discount. Abad and Aggarwal [1] elaborated a model to determine the buyer's lot size and price with all-unit quantity discount under shipment sizes. Leaveano, Jafar, Saleh, Muhammad, and Rahman [22], and Nie, Xu, and Zhan [26] introduced an integrated inventory model by incorporating transportation cost. However their model did not considered stochastic demand. Jauhari, Fitriyani, and Aisyati [18] studied stochastic demand and freight rate discounts in their model. Sarkar, Ganguly, Sarkar, and Pareek [28] examined an integrated inventory model by considering variable transportation and carbon emission costs. Modelling freight rates is estimating freight rates based on the value of some parameters in a continuous function. Examples of these parameters include: (a) the TL charge in an inverse function (Swenseth & Godfrey, [32]; Yildirmaz, Karabati, & Sayin, [33]); (b) the distance in a proportional function (Ballou, [2]); (c) the constant used in an exponential function (Buffa, [6], [7]); (d) the smoothing constant in an adjusted inverse function (Swenseth & Buffa, [29], [30]; Swenseth & Godfrey, [31], [32]); and (e) load density, shipment weight, and shipment distance in a nonlinear model (Kay & Warsing, [20]). Ballou [2] argued that time, effort, and cost considerations often indicate the logistic decisions. And finally Ivan Darma Wangsa & Hui Ming Wee [17] presented an integrated vendor-buyer inventory model with transportation cost and stochastic demand.

This paper is an extension of "An integrated vendor-buyer inventory model with transportation cost and stochastic demand" by Ivan Darma Wangsa & Hui Ming Wee [17]. Here we considered that the lead time is proportional to the batch size Q and in addition to other delays such as transportation time, by relaxing the assumption of constant lead time.

3. Notations and Assumptions

To establish the mathematical model, the following notations and assumptions are used.

Decision Variables

Q Order quantity of the buyer.

k Safety factor of the buyer.

m The number of shipments of the product delivered from the vendor to the buyer in one production cycle, a positive integer.

r Buyer's reorder point.

Parameters

D Average demand per unit time on the buyer.

P Production rate of the vendor ($P > D$).

A Buyer's ordering cost per order.

S Vendor's setup cost per setup.

C_b Unit purchase cost paid by the buyer.

C_v Unit production cost incurred by the vendor, $C_v < C_b$.

r_b Buyer's holding cost rate per unit time.

r_v Vendor's holding cost rate per unit time.

$L(Q)$ Lead time = $\frac{Q}{P} + b$, where b denotes a fixed delay due to transportation, production time of other products scheduled during the lead time on the same facility.

F_0 Vendor's transportation cost per trip.

w Weight of a unit part (lbs per unit).

d Transportation distance (miles).

α Discount factor for LTL shipments, $0 \leq \alpha \leq 1$.

F_x The freight rate in dollar per pound for a given per mile for full truckload (FTL).

F_y The freight rate in dollar per pound for a given per for partial load.

W_x Full truckload (FTL) shipping weight (lbs).

W_y Actual shipping weight (lbs).

π_x Backorder cost per unit of the buyer.

π_0 Marginal profit per unit of the buyer.

β The backorder ratio, $0 \leq \beta \leq 1$.

$B(r, L(Q))$ Expected demand shortage at the end of cycle.

X The lead time demand, which follows a normal distribution with finite mean $DL(Q)$ and standard deviation $\sigma\sqrt{L(Q)}$, where σ denotes the standard deviation of demand per unit time, $X \sim N(DL(Q), \sigma\sqrt{L(Q)})$.

$E(.)$ Mathematical expectation.

x^+ Maximum value of x and 0 . i.e. $x^+ = \max\{x, 0\}$.

b Fixed delay due to transportation.

The following assumptions are used in our model:

1. A single item is considered with a single vendor and a single buyer.

2. Production rate P is finite. ($P > D$).

3. The vendor manufactures mQ with a finite production rate P in one setup and ships quantity Q to the buyer over m times.

4. The demand X during lead time $L(Q)$ follows a normal distribution with mean $DL(Q)$ and standard deviation $\sigma\sqrt{L(Q)}$.

5. Shortages are permitted with partial backorders and lost sales.

6. All items are purchased Free-On-Board (FOB). All the transportation charges are incurred by the buyer.

7. The reorder point $r = \text{Expected demand during lead time} + \text{Safety stock} = DL(Q) + k\sigma\sqrt{L(Q)}$ where $k\sigma\sqrt{L(Q)}$ is the safety stock and k is the safety factor.

4. Model Formulation

A single-vendor single-buyer supply chain is considered with freight forwarding. The vendor produces the product in a batch size of mQ with a finite production rate P , $P > D$. The quantity Q is shipped to the buyer over m times. The vendor incurs a transportation cost for delivering to the freight forwarding is F_0 . He delivers to the freight forwarding who will make amalgamation and delivery to the buyer with freight rate (F_x). The freight rates are functions of shipping weight (W_x), distance (d) and transportation modes. The inventory policy with variable lead time is considered for the buyer. We assume that lead time is proportional to the lot size produced by the vendor in addition to a fixed delay due to transportation, non productive time, that is $L(Q) = \frac{Q}{P} + b$.

The estimated total freight cost per year as the function of shipping weight and distance with adjusted inverse yields (Leaveano, Jafar, Saleh, et al., [22]; Nie et al., [26]) is given as

$$F(D, Q, w, d) = \frac{D}{Q} \alpha F_x W_x d + Ddw(1 - \alpha)F_x$$

The total relevant cost for the buyer (TRC_b) is given by

$TRC_b = \text{Ordering cost} + \text{holding cost} + \text{shortage cost} + \text{freight cost.}$

$TRC_b(Q, k, r)$

$$\begin{aligned} &= \frac{D}{Q} A + r_b C_b \left[\frac{Q}{2} + k\sigma\sqrt{L(Q)} + (1 - \beta)B(r, L(Q)) \right] \\ &+ \frac{D}{Q} [\pi_x \beta + \pi_0(1 - \beta)]B(r, L(Q)) + F(D, Q, w, d) \end{aligned}$$

Where $B(r, L(Q)) = \int_r^\infty (x - r)f(x, DL(Q), \sigma\sqrt{L(Q)}) dx$

$$= \sigma\sqrt{L(Q)}\psi(k)$$

$$= \sigma\sqrt{\frac{Q}{P} + b}\psi(k)$$

(1)

and $\psi(k) = \int_k^\infty (z - k)\Phi(z)dz$

$$= \varphi(k) - k(1 - \Phi(k))$$

(2)

where φ , Φ are standard normal pdf and distribution function (df), respectively.

and $k = (r - DL(Q))/\sigma\sqrt{L(Q)}$ (3)

The total relevant cost for the vendor (TRC_v) is given by

$TRC_v =$ setup cost + transportation cost + holding cost

$$TRC_v(Q, m) = \frac{D}{mQ}S + F_0 \frac{D}{Q} + r_v C_v \frac{Q}{2} \left[m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right]$$

Therefore, the joint total relevant cost of vendor and buyer is given by

$$JTRC(Q, k, r, m) = TRC_b(Q, k, r) + TRC_v(Q, m)$$

$$\begin{aligned} JTRC(Q, k, r, m) &= \frac{D}{Q} \{ A + [\pi_x \beta + \pi_0(1 - \beta)] B(r, L(Q)) + \alpha F_x W_x d \} \\ &+ r_b C_b \left[\frac{Q}{2} + k \sigma \sqrt{L(Q)} + (1 - \beta) B(r, L(Q)) \right] + Ddw(1 - \alpha) F_x + \frac{D}{Q} \left(\frac{S}{m} + F_0 \right) \\ &+ r_v C_v \frac{Q}{2} \left[m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] \end{aligned} \tag{4}$$

To simplify the notation, let

$$G(m) = A + \frac{S}{m} + F_0 + \alpha F_x W_x d$$

$$H(m) = r_b C_b + r_v C_v \left[m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right]$$

Consequently, the expression of the joint total relevant cost can be rewritten as

$$\begin{aligned} JTRC(Q, k, m) &= \frac{G(m)D}{Q} + \frac{Q}{2} H(m) + \frac{D}{Q} [\pi_x \beta + \pi_0(1 - \beta)] \sigma \sqrt{\frac{Q}{P} + b} \psi(k) + r_b C_b k \sigma \sqrt{\frac{Q}{P} + b} \\ &+ r_b C_b (1 - \beta) \sigma \sqrt{\frac{Q}{P} + b} \psi(k) + Ddw(1 - \alpha) F_x \end{aligned} \tag{5}$$

For fixed m , let us take the first partial derivatives of $JTRC(Q, k, m)$ with respect to Q and k and setting them to zero, we get

$$Q = \sqrt{\frac{2D \left\{ G(m) + [\pi_x \beta + \pi_0(1 - \beta)] \sigma \psi(k) \sqrt{\frac{Q}{P} + b} \right\}}{\frac{[\pi_x \beta + \pi_0(1 - \beta)] D \sigma \psi(k)}{PQ \sqrt{\frac{Q}{P} + b}} + H(m) + \frac{r_b C_b [k \sigma + (1 - \beta) \sigma \psi(k)]}{P \sqrt{\frac{Q}{P} + b}}} \tag{6}$$

$$\phi(k) = 1 - \frac{r_b C_b Q}{D[\pi_x \beta + \pi_0(1 - \beta)] + r_b C_b Q(1 - \beta)} \tag{7}$$

The following iterative procedure can be used to find an approximate solution to the above problem.

Algorithm:

Step 0: Set $JTRC^* = \infty$ and $m = 1$

Step 1: Compute $Q = \left\lceil \sqrt{2DG(m)/H(m)} \right\rceil$ where $[x]$ the nearest integer of x .

Step 2:

- Find k from (7)
- Compute $\psi(k)$ using (2)

Step 3:

- Compute Q' using (6)
- Set $Q' = [Q']$

Step 4:

- If $|Q' - Q| = 0$, compute $JTRC(Q, k, m)$ and go to Step 5.
- If $|Q' - Q| > 0$, set $Q \leftarrow Q'$ and go to Step 2.

Step 5:

- If $JTRC^* \geq JTRC(Q, k, m)$ then $JTRC^* \leftarrow JTRC(Q, k, m)$, $Q^* \leftarrow Q$, $r^* \leftarrow r$, $k^* \leftarrow k$, Set $m \leftarrow m + 1$ and go to Step 1
- Otherwise $m^* \leftarrow m - 1$ and stop.

5. Numerical example

In this section we provide the numerical example to illustrate the above solution procedure. The following parameters are used for finding the result:

$D = 10,000$ units/year, $P = 40,000$ units/year, $\sigma = 7$ units/week, $A = \$30$ /order, $S = \$3600$ /setup, $r_b = 0.2$, $r_v = 0.2$, $C_b = \$225$ /unit, $C_v = \$190$ /unit, $F_0 = \$50$ /trip, $\pi_x = \$100$ /unit, $\pi_0 = \$300$ /unit, $\beta = 0.25$, $\alpha = 0.11246$, $w = 22$ lbs/unit, $d = 600$ miles, $F_x = \$0.0000402174$ /lb/mile, $W_x = 46,000$ lbs and $b = 0.01$.

From table 1, we have

The optimum number of shipments $m^* = 4$

The optimum order quantity $Q^* = 397$

The optimum reorder point $r^* = 202$

The optimum safety factor $k^* = 2.45$ and $JTRC^* = 60454.80$

The following table shows the optimal number of shipments, safety factor, reorder point and batch quantity.

Table 1.

m	k	Q	r	JTRC
1	2.03	1181	398	69261.55
2	2.24	695	276	62535.72
3	2.36	502	228	60810.65
4	2.45	397	202	60454.80
5	2.52	331	185	60659.75

6. Conclusion

In this paper, we considered the single vendor single buyer integrated production inventory problem with stochastic demand and freight forwarding. Instead of constant lead time we assumed that the lead time is variable and depends on the batch size and other delays, such as transportation time. A simple procedure is suggested to obtain an approximate solution of the proposed model.

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